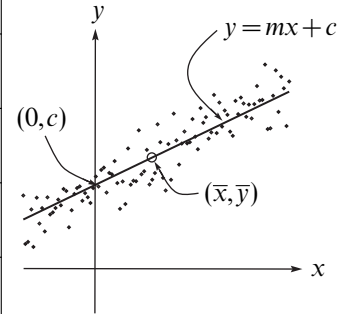


2.14 Numerical methods

Straight-line fitting^a

Data	$(\{x_i\}, \{y_i\})$ n points	(2.570)
Weights ^b	$\{w_i\}$	(2.571)
Model	$y = mx + c$	(2.572)
Residuals	$d_i = y_i - mx_i - c$	(2.573)
Weighted centre	$(\bar{x}, \bar{y}) = \frac{1}{\sum w_i} \left(\sum w_i x_i, \sum w_i y_i \right)$	(2.574)
Weighted moment	$D = \sum w_i (x_i - \bar{x})^2$	(2.575)
Gradient	$m = \frac{1}{D} \sum w_i (x_i - \bar{x}) y_i$	(2.576)
	$\text{var}[m] \simeq \frac{1}{D} \frac{\sum w_i d_i^2}{n-2}$	(2.577)
Intercept	$c = \bar{y} - m\bar{x}$	(2.578)
	$\text{var}[c] \simeq \left(\frac{1}{\sum w_i} + \frac{\bar{x}^2}{D} \right) \frac{\sum w_i d_i^2}{n-2}$	(2.579)



^aLeast-squares fit of data to $y = mx + c$. Errors on y -values only.

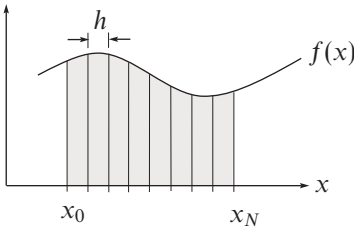
^bIf the errors on y_i are uncorrelated, then $w_i = 1/\text{var}[y_i]$.

Time series analysis^a

Discrete convolution	$(r \star s)_j = \sum_{k=-(M/2)+1}^{M/2} s_{j-k} r_k$	(2.580)	r_i response function s_i time series M response function duration
Bartlett (triangular) window	$w_j = 1 - \left \frac{j - N/2}{N/2} \right $	(2.581)	w_j windowing function N length of time series
Welch (quadratic) window	$w_j = 1 - \left[\frac{j - N/2}{N/2} \right]^2$	(2.582)	The graph shows four bell-shaped curves representing different windowing functions. The x-axis is labeled j/N and ranges from 0 to 1. The y-axis is labeled w and ranges from 0 to 1. The curves are labeled: Welch (quadratic), Bartlett (triangular), Hanning, and Hamming. All curves are symmetric and peak at $j/N = 0.5$.
Hanning window	$w_j = \frac{1}{2} \left[1 - \cos \left(\frac{2\pi j}{N} \right) \right]$	(2.583)	
Hamming window	$w_j = 0.54 - 0.46 \cos \left(\frac{2\pi j}{N} \right)$	(2.584)	

^aThe time series runs from $j = 0 \dots (N-1)$, and the windowing functions peak at $j = N/2$.

Numerical integration

		$h = (x_N - x_0)/N$ (subinterval width) $f_i = f(x_i)$ N = number of subintervals
Trapezoidal rule	$\int_{x_0}^{x_N} f(x) dx \simeq \frac{h}{2} (f_0 + 2f_1 + 2f_2 + \cdots + 2f_{N-1} + f_N) \quad (2.585)$	
Simpson's rule ^a	$\int_{x_0}^{x_N} f(x) dx \simeq \frac{h}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots + 4f_{N-1} + f_N) \quad (2.586)$	

^a N must be even. Simpson's rule is exact for quadratics and cubics.

Numerical differentiation^a

$\frac{df}{dx} \simeq \frac{1}{12h} [-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)]$	(2.587)
$\sim \frac{1}{2h} [f(x+h) - f(x-h)]$	(2.588)
$\frac{d^2f}{dx^2} \simeq \frac{1}{12h^2} [-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)]$	(2.589)
$\sim \frac{1}{h^2} [f(x+h) - 2f(x) + f(x-h)]$	(2.590)
$\frac{d^3f}{dx^3} \sim \frac{1}{2h^3} [f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$	(2.591)

^aDerivatives of $f(x)$ at x . h is a small interval in x .

Relations containing " $\simeq$ " are $O(h^4)$; those containing " $\sim$ " are $O(h^2)$.

Numerical solutions to $f(x)=0$

Secant method	$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) \quad (2.592)$	f = function of x x_n = $f(x_\infty)=0$ f' = df/dx
Newton-Raphson method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (2.593)$	



Numerical solutions to ordinary differential equations^a

Euler's method	if	$\frac{dy}{dx} = f(x, y)$	(2.594)
	and	$h = x_{n+1} - x_n$	(2.595)
	then	$y_{n+1} = y_n + hf(x_n, y_n) + O(h^2)$	(2.596)
Runge–Kutta method (fourth-order)	if	$\frac{dy}{dx} = f(x, y)$	(2.597)
	and	$h = x_{n+1} - x_n$	(2.598)
		$k_1 = hf(x_n, y_n)$	(2.599)
		$k_2 = hf(x_n + h/2, y_n + k_1/2)$	(2.600)
		$k_3 = hf(x_n + h/2, y_n + k_2/2)$	(2.601)
		$k_4 = hf(x_n + h, y_n + k_3)$	(2.602)
	then	$y_{n+1} = y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5)$	(2.603)

^aOrdinary differential equations (ODEs) of the form $\frac{dy}{dx} = f(x, y)$. Higher order equations should be reduced to a set of coupled first-order equations and solved in parallel.

